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ELECTRICAL ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

CONTROL SYSTEMS

1 CHAPTER Introduction

Q.1 (a) A control system is defined by following mathematical relationship

$$\frac{d^2x}{dt^2}+\frac{6dx}{dt}+5x=12(1-e^{-2t})$$

Find the response of the system at $t \to \infty$

(b) A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt}+y(t)=\delta(t)$$

Where $\delta(t)$ is delta function. Assuming zero initial condition and denoting unit step function by $u(t)$. Find $y(t)$.

Solution:

(a) Taking LT on both sides

$$(s^2+6s+5)\,X(s) = 12\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$$(s+1)\,(s+5)\,X(s) = \frac{24}{s(s+2)}$$

$$X(s) = \frac{24}{s(s+1)\,(s+2)\,(s+5)}$$

Response at $t \to \infty$

Using final value theorem,

$$\boxed{\lim_{t\to\infty} x(t) = \lim_{s\to 0}[sX(s)]} = \lim_{s\to 0}\frac{s\times 24}{s(s+1)\,(s+2)\,(s+5)} = 2.4$$

(b) Taking Laplace transform on both sides

$$Y(s)\,[s+1] = 1$$

$$Y(s) = \frac{1}{s+1}$$

By taking inverse Laplace transform

$$y(t) = e^{-t}\,u(t)$$

Q.2 (a) The Laplace equation for the charging current, $i(t)$ of a capacitor arranged in series with a resistance is given by

$$I(s) = \frac{sC}{1+sRC}\cdot E(s)$$

The circuit is connected to a supply voltage of E. If E = 100 V, R = 2 MΩ, C = 1 μF. Calculate the initial value of the charging current.

(b) A series circuit consisting of resistance R and an inductance of L is connected to a d.c. supply voltage of E. Derive an expression for the steady-state value of the current flowing in the circuit using final value theorem.

Solution:

(a) Since, $E = 100\, v(t)$

Taking Laplace Transform, $E = 100\,(t)$ volts,

$$\therefore \quad E(s) = \frac{100}{s}$$

Substituting the given values,

$$I(s) = \frac{1\times 10^{-6}s}{(2\times 10^{6}\times 1\times 10^{-6}s+1)}\cdot\frac{100}{s} = \frac{10^{-6}s}{2s+1}\cdot\frac{100}{s}$$

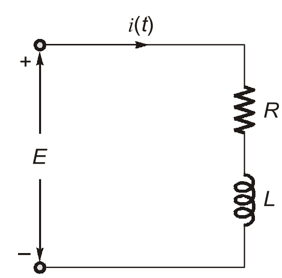
Applying the initial value theorem,

$$\boxed{i(0^+) = \lim_{t\to 0} i(t) = \lim_{s\to\infty} s\,I(s)}$$

$$i(0^+) = \lim_{s\to\infty} s\cdot\frac{10^{-4}}{1+2s} = \lim_{s\to\infty}\cdot\frac{10^{-4}}{\frac{1}{s}+2} = 50\ \mu\text{A}$$

(b) The differential equation relating the current $i(t)$ flowing in the circuit and the input voltage E is given by

$$E = R\,i(t) + L\frac{di(t)}{dt}$$



Taking Laplace transform of the equation yields,

$$E(s) = R\,I(s) + L[(sI(s) - i(0^+))]$$

Assume, $i(0^+) = 0$

$$\therefore \quad E(s) = R\,I(s) + Ls\,I(s)$$

$\because$ E is constant (d.c. voltage)

$$E(s) = \frac{E}{s} = RI(s) + Ls\,I(s)$$

$$I(s) = \frac{E}{s(R+sL)}$$

Applying the final value theorem,

$$i_{ss} = \lim_{t\to\infty} i(t) = \lim_{s\to 0} s\,I(s) = \lim_{s\to 0}\frac{sE}{s(R+sL)}$$

$$i_{ss} = \frac{E}{R}$$

Q.3 **The impulse response of a system S_1 is given by $y_1(t) = 4e^{-2t}$. The step response of a system S_2 is given by $y_2(t) = 2(1 - e^{-3t})$. The two systems are cascaded together without any interaction. Find response of the cascaded system for unit ramp input.**

Solution:

(a) Taking the Laplace transform of the response of S_1, we get

$$Y_1(s) = \frac{4}{s+2},$$

$$X_1(s) = 1 \;\ldots\; (x(t) = \delta(t))$$

Therefore, $$G_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{4}{s+2} \qquad [\because Y_1(s) = 1]$$

Taking the Laplace transform of the response of S_2, we get

$$Y_2(s) = 2\left(\frac{1}{s} - \frac{1}{s+3}\right) = \frac{6}{s(s+3)}$$

$$Y_2(s) = \frac{1}{s} \; \dots \; (x_2(t) = u(t))$$

Thus, $$G_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{6}{s(s+3)} \cdot s = \frac{6}{s+3}$$

(b) The transfer function of the cascaded system is

$$G(s) = G_1(s)G_2(s) = \frac{24}{(s+2)(s+3)}$$

The Laplace transform of unit ramp is $R(s) = \frac{1}{s^2}$. Therefore,

$$G(s) = \frac{C(s)}{R(s)}$$

$$C(s) = \frac{24}{(s+2)(s+3)} \cdot \frac{1}{s^2}$$

$$\equiv \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \left.\frac{24}{(s+2)(s+3)}\right|_{s=0} = 4$$

$$B = \frac{d}{ds}\left[s^2 C(s)\right]_{s=0}$$

$$= \frac{d}{ds}\left[\frac{24}{(s+2)(s+3)}\right] = -\left.\frac{24(2s+5)}{(s+2)^2(s+3)^2}\right|_{s=0}$$

$$= -\frac{10}{3}$$

$$C = \left.\frac{24}{s^2(s+3)}\right|_{s=-2} = 6$$

$$D = \left.\frac{24}{s^2(s+2)}\right|_{s=-3} = -\frac{8}{3}$$

$$C(s) = \frac{4}{s^2} - \frac{10}{3}s + \frac{6}{s+2} - \frac{8}{3}e^{-3t}$$

Taking inverse Laplace transform.

Therefore, $$c(t) = 4t - \frac{10}{3}u(t) + 6e^{-3t} - \frac{8}{3}e^{-3t}$$

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2
CHAPTER

Transfer Function

Q.1 A torque T N-m is applied to a shaft having a moment of inertia J and coefficient of viscous friction of f produces an angular shift of θ radius. Obtain the transfer function in relation to θ and T.

Solution:

The equation for the system is given by

$$T = \frac{Jd^2\theta}{dt^2} + f\frac{d\theta}{dt} \quad \text{...(1)}$$

Assuming initial conditions as zero and taking the Laplace transform on both sides of equation (1), the following equations is obtained,

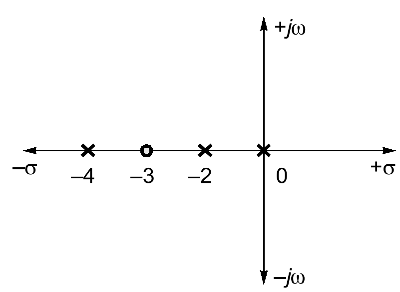
$$T(s) = Js^2\,\theta(s) + fs\,\theta(s)$$

$$T(s) = s(Js + f))\,\theta(s) \quad \text{...(2)}$$

From equation (2), the required transfer function is obtained below,

$$\frac{\theta(s)}{T(s)} = \frac{1}{s(Js+f)}$$

Q.2 The pole-zero configuration of a transfer function is given below. The value of the transfer function as s = 1 is found to be 3.2. Determine the transfer function and gain factor K.



Solution:

The transfer function has three poles and one zero therefore, the transfer function consists of one term in the numerator and three terms in the denominator.

Poles are located at $s = 0$, $s = -2$, $s = -4$

Zeros are located at $s = -3$

The transfer function, $$G(s) = \frac{K(s+3)}{s(s+2)(s+4)}$$

It is given that at, $s = 1$, $G(s) = 3.2$

$$\Rightarrow \quad 3.2 = \frac{4K}{15} \Rightarrow K = 12$$

$$\therefore \quad G(s) = \frac{12(s+3)}{s(s+2)(s+4)}$$